2차원 Viewing 변환

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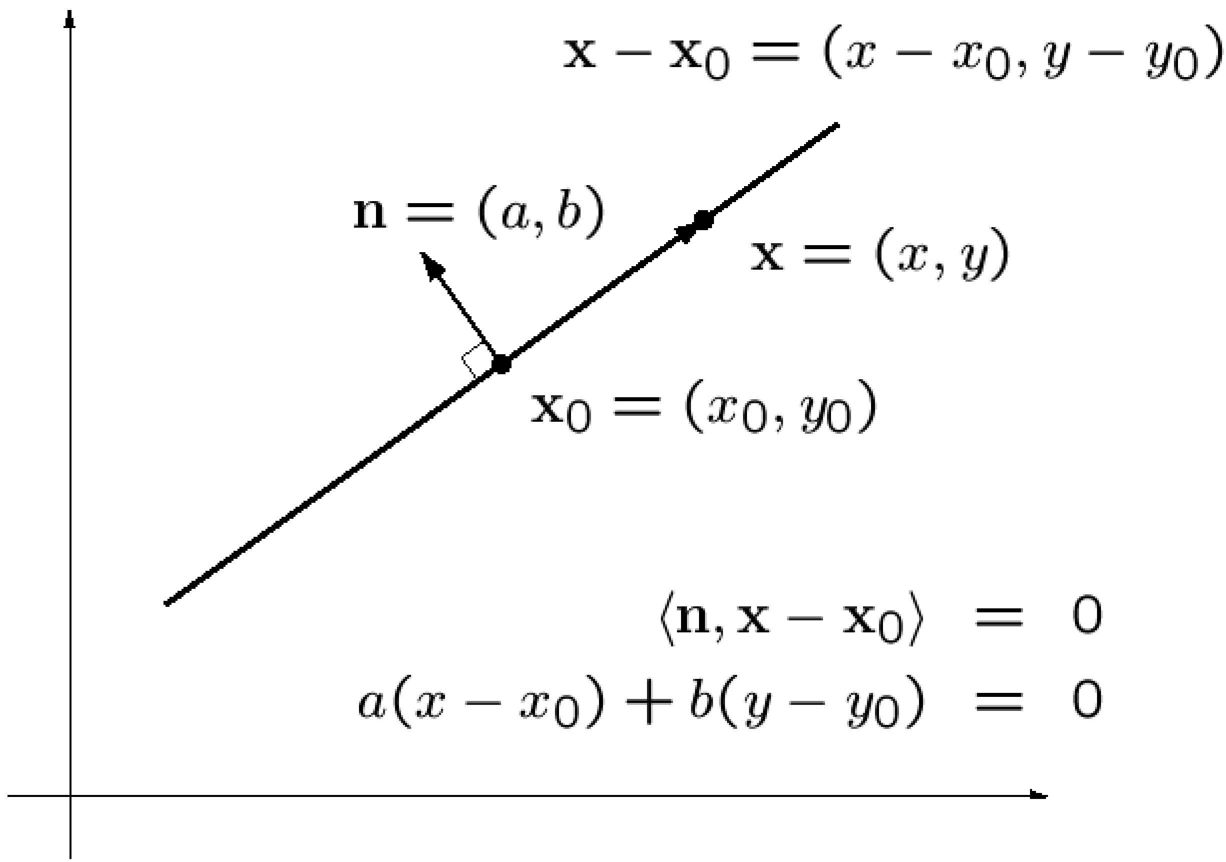
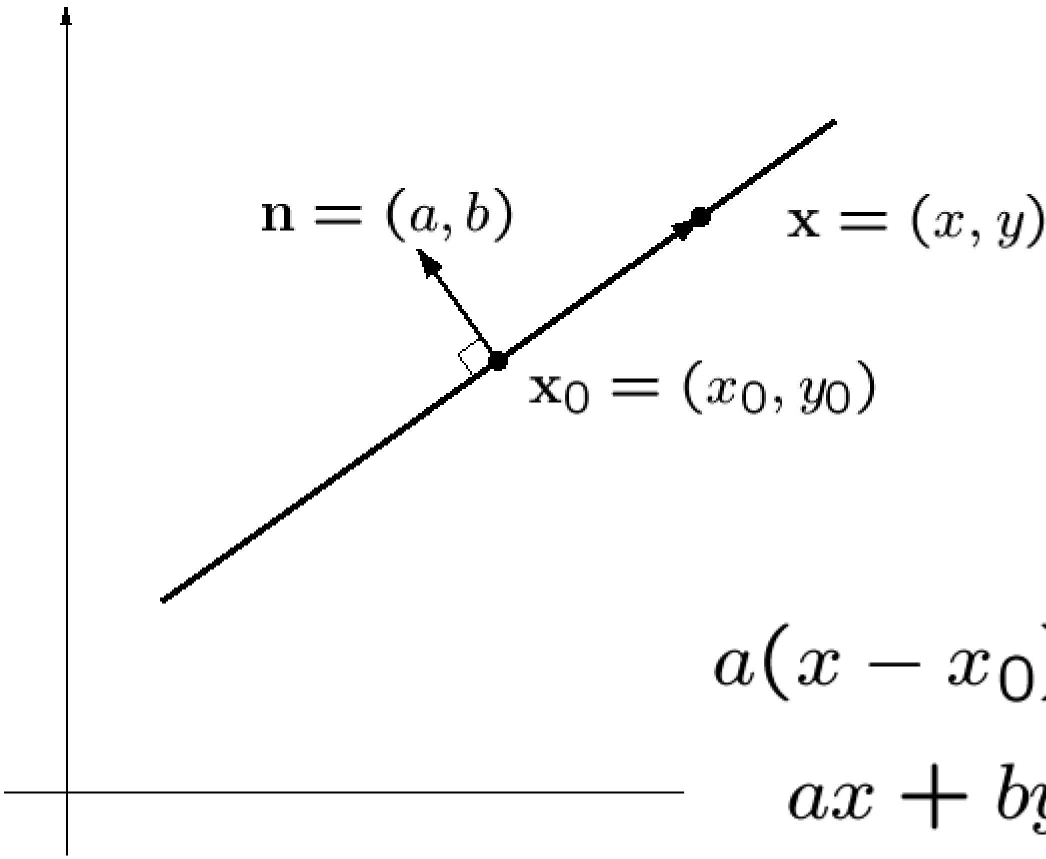
직선의 방정식



$\mathbf{x} - \mathbf{x}_0 = (x - x_0, y - y_0)$
$\mathbf{n} = (a, b)$
$\mathbf{x} = (x, y)$
$\mathbf{x}_0 = (x_0, y_0)$
$\langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle = 0$
$a(x - x_0) + b(y - y_0) = 0$

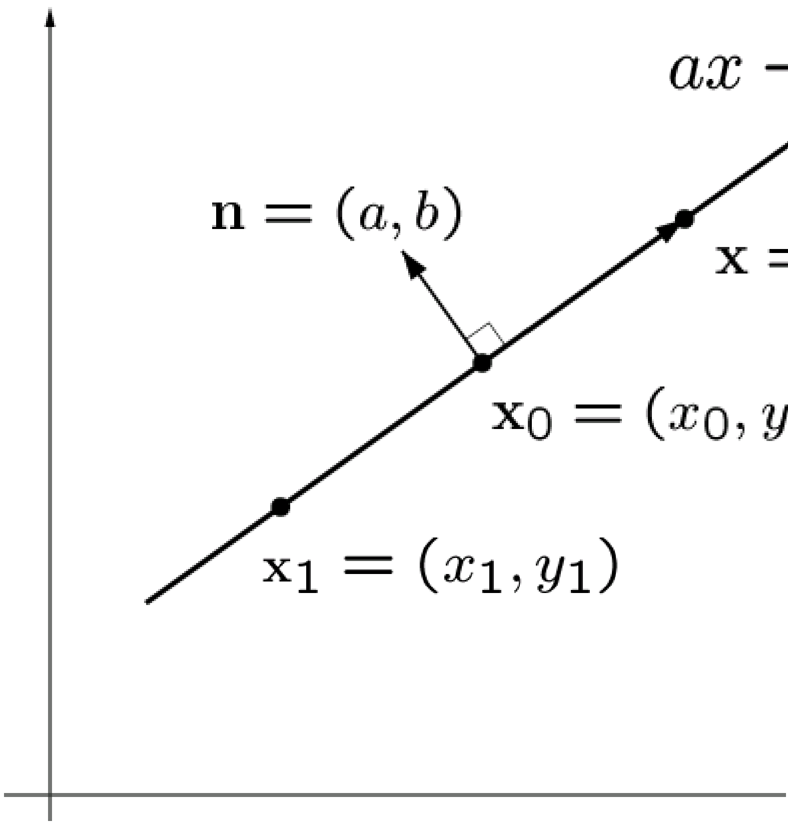
직선의 방정식



$$
\begin{aligned}
\langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle &= 0 \\
a(x - x_0) + b(y - y_0) &= 0 \\
ax + by - ax_0 - by_0 &= 0 \\
ax + by + c &= 0
\end{aligned}
$$

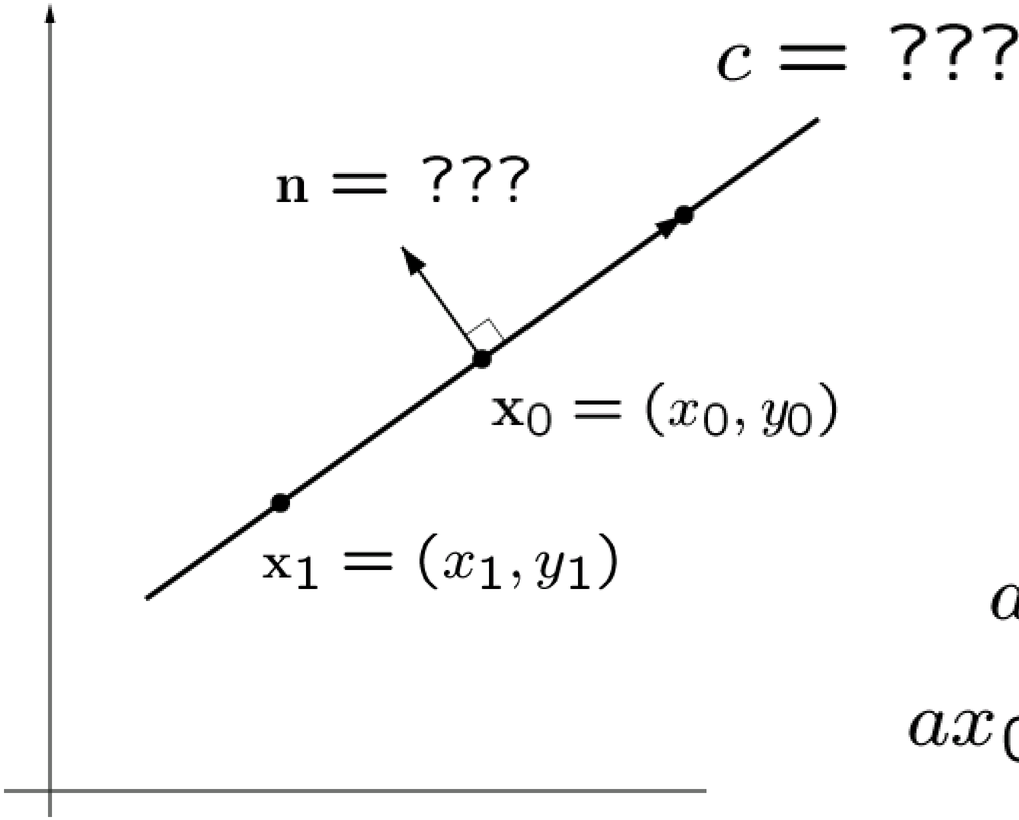
직선의 방정식

$$ax + by + c = 0$$



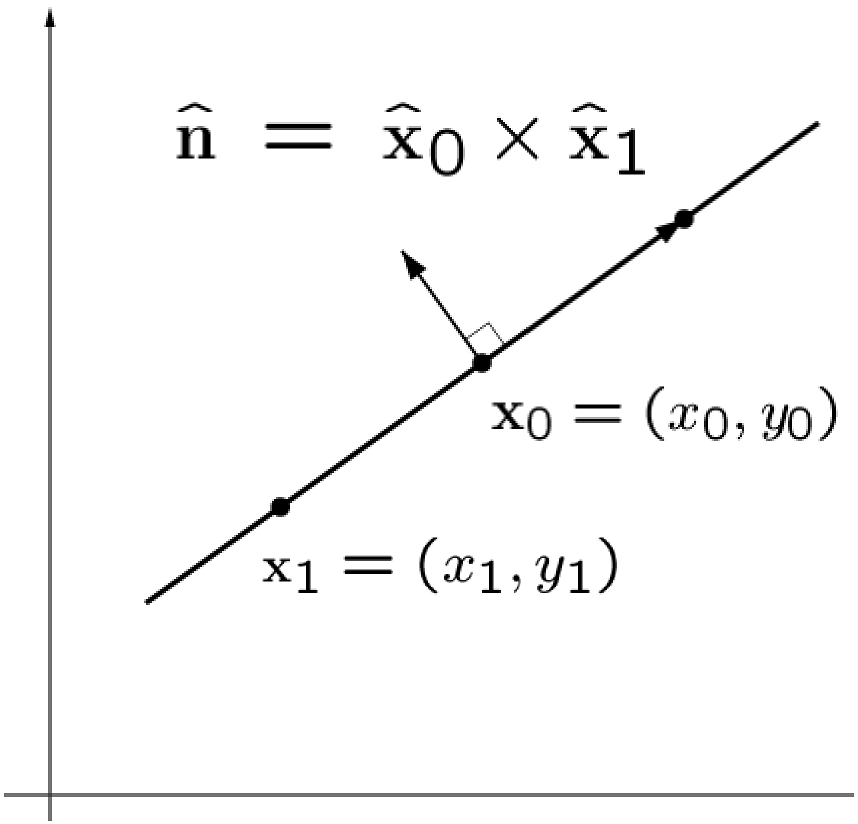
$$\begin{aligned}
ax + by - ax_0 - by_0 &= 0 \\
ax + by + c &= 0 \\
ax_0 + by_0 + c \cdot 1 &= 0 \\
ax_1 + by_1 + c \cdot 1 &= 0
\end{aligned}$$

두점으로 부터 직선구하기



$$ax + by + c \cdot 1 = 0$$
$$ax_0 + by_0 + c \cdot 1 = 0$$
$$ax_1 + by_1 + c \cdot 1 = 0$$

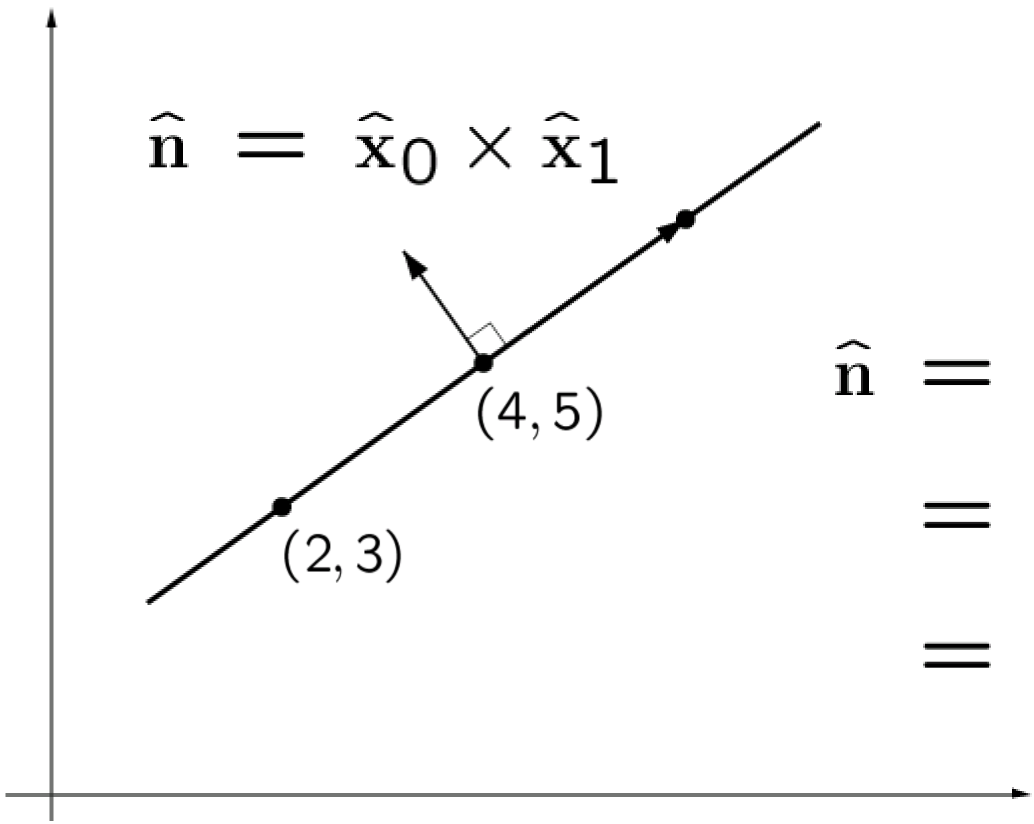
두점으로 부터 직선구하기



$$\hat{\mathbf{n}} = (a, b, c)$$
$$\hat{\mathbf{x}}_0 = (x_0, y_0, 1)$$
$$\hat{\mathbf{x}}_1 = (x_1, y_1, 1)$$

$$ax_0 + by_0 + c \cdot 1 = 0$$
$$ax_1 + by_1 + c \cdot 1 = 0$$
$$\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}_0 \rangle = 0$$
$$\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}_1 \rangle = 0$$

두점으로 부터 직선구하기

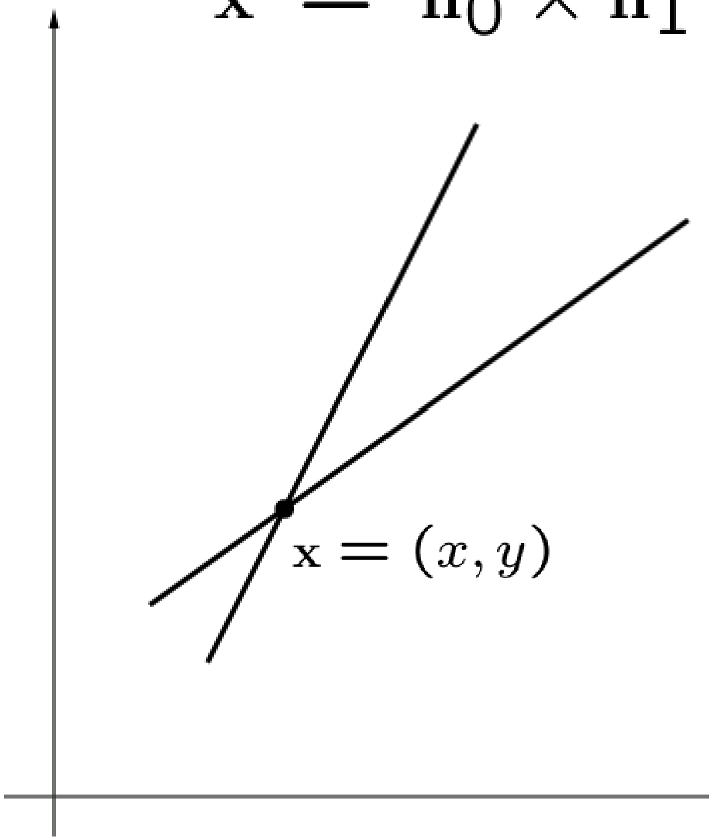


$$\hat{\mathbf{x}}_0 = (2, 3, 1)$$
$$\hat{\mathbf{x}}_1 = (4, 5, 1)$$

$$\begin{aligned}\hat{\mathbf{n}} &= \hat{\mathbf{x}}_0 \times \hat{\mathbf{x}}_1 \\ &= (2, 3, 1) \times (4, 5, 1) \\ &= (-2, 2, -2)\end{aligned}$$

두직선으로 부터 교점구하기

$$\hat{\mathbf{x}} = \hat{\mathbf{n}}_0 \times \hat{\mathbf{n}}_1$$



$$\hat{\mathbf{x}} = (x, y, 1)$$

$$\hat{\mathbf{n}}_0 = (a_0, b_0, c_0)$$

$$\hat{\mathbf{n}}_1 = (a_1, b_1, c_1)$$

$$a_0 \cdot x + b_0 \cdot y + c_0 \cdot 1 = 0$$

$$a_1 \cdot x + b_1 \cdot y + c_1 \cdot 1 = 0$$

$$\langle \hat{\mathbf{n}}_0, \hat{\mathbf{x}} \rangle = 0$$

$$\langle \hat{\mathbf{n}}_1, \hat{\mathbf{x}} \rangle = 0$$

두직선으로 부터 교점구하기

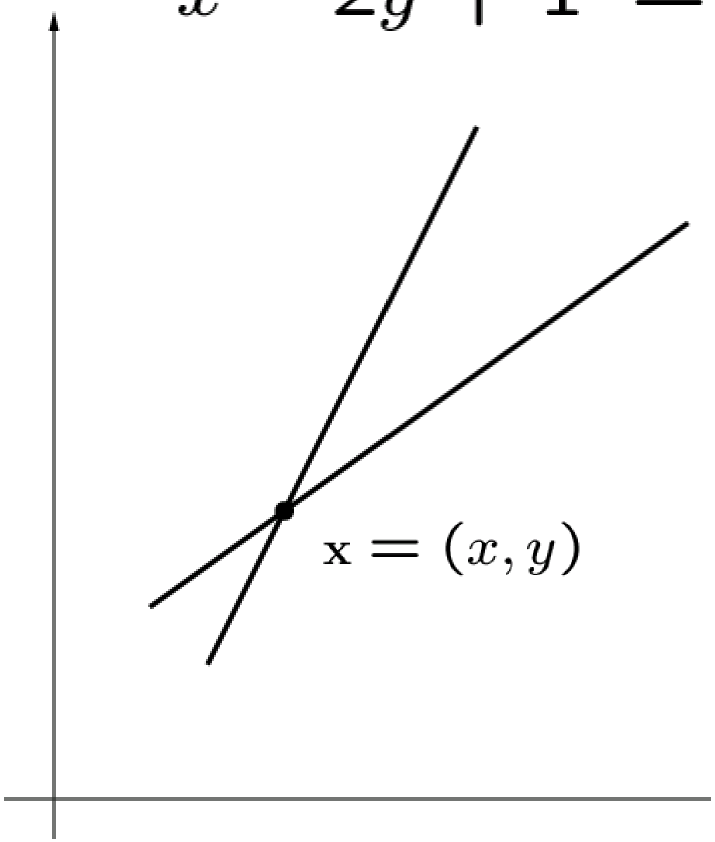
$$x + y - 1 = 0$$

$$x - 2y + 1 = 0$$

$$\hat{\mathbf{x}} = (x, y, 1)$$

$$\hat{\mathbf{n}}_0 = (1, 1, -1)$$

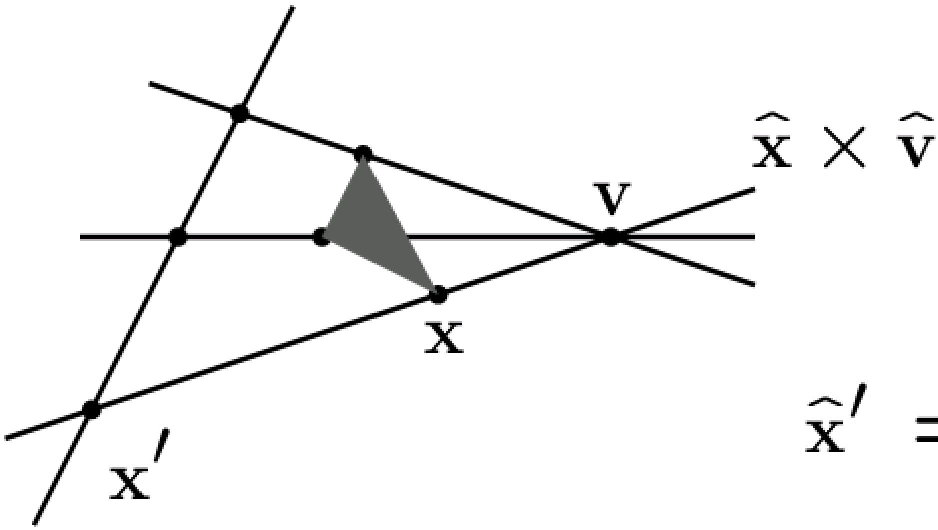
$$\hat{\mathbf{n}}_1 = (1, -2, 1)$$



$$\begin{aligned} \hat{\mathbf{x}} &= \hat{\mathbf{n}}_0 \times \hat{\mathbf{n}}_1 \\ &= (1, 1, -1) \times (1, -2, 1) \\ &= (-1, -2, -3) \\ &= \left(\frac{1}{3}, \frac{2}{3}, 1\right) \end{aligned}$$

2차원에서의 투영변환

$$\hat{\mathbf{n}} = (a, b, c)$$



$$\hat{\mathbf{x}}' = (x', y', 1)$$
$$= \hat{\mathbf{n}} \times (\hat{\mathbf{x}} \times \hat{\mathbf{v}})$$
$$= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \hat{\mathbf{v}}$$
$$= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\hat{\mathbf{x}} = (x, y, 1)$$
$$\hat{\mathbf{v}} = (v_x, v_y, 1)$$

2차원에서의 투영변환

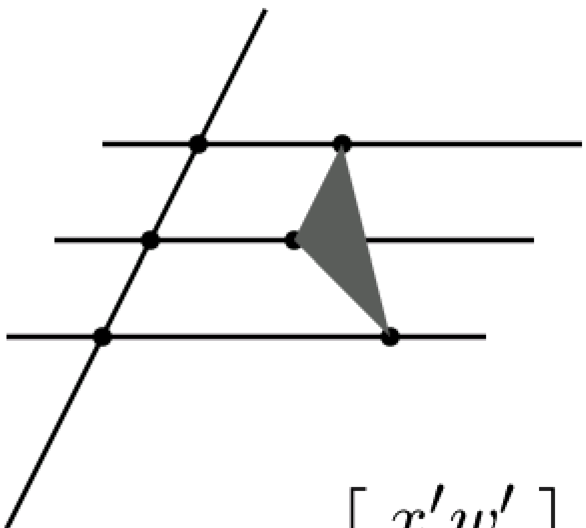
$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} bv_y + c & -bv_x & -cv_x \\ -av_y & av_x + c & -cv_y \\ -a & -b & av_x + bv_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2차원에서의 평행투영변환



$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\longleftarrow \quad \hat{\mathbf{v}} = (v_x, v_y, 0)$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} - \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2차원에서의 평행투영변환

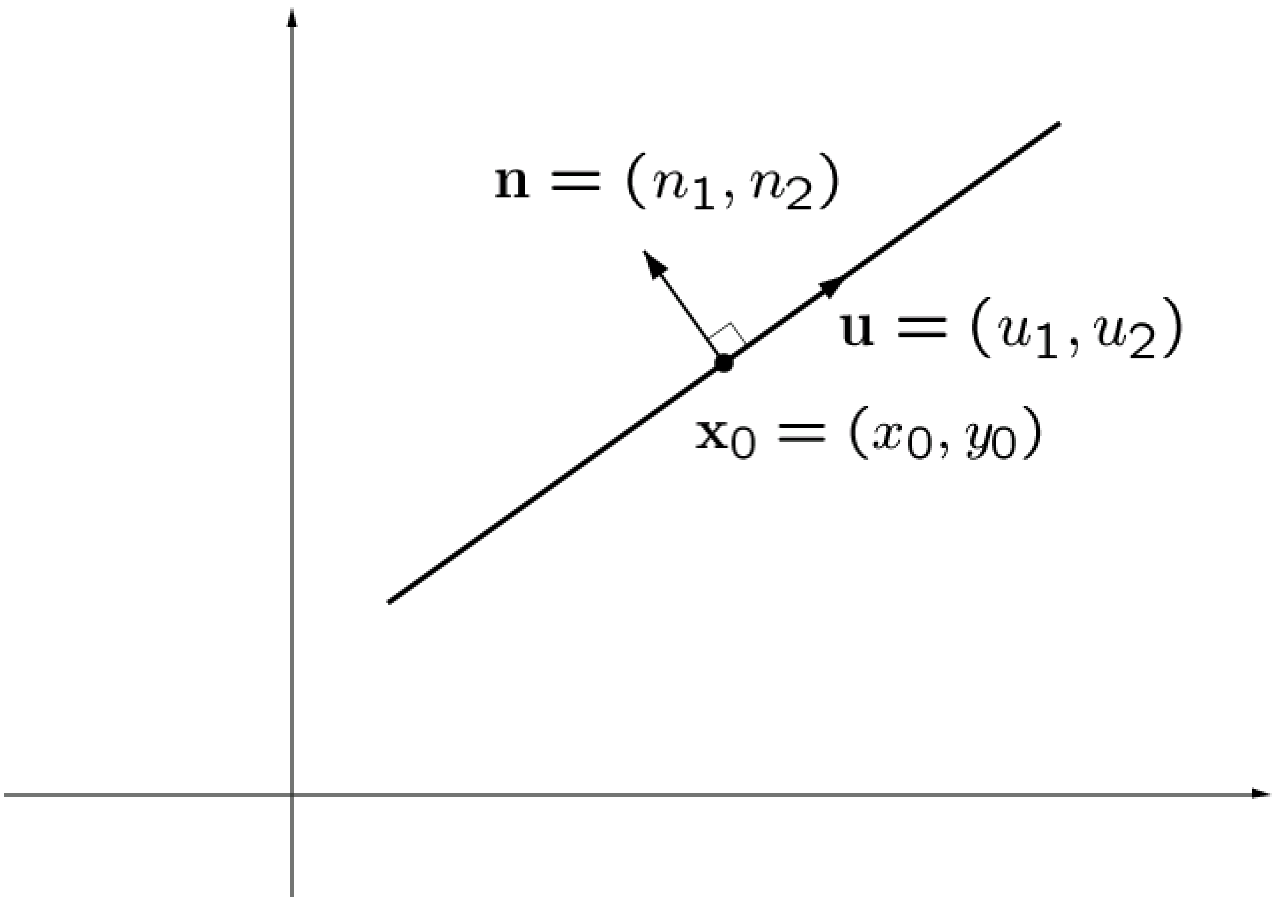
$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} - \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} bv_y & -bv_x & -cv_x \\ -av_y & av_x & -cv_y \\ 0 & 0 & av_x + bv_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2차원에서의 Viewing 변환

$$\|\mathbf{u}\| = \|\mathbf{n}\| = 1$$



$$\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2차원에서의 Viewing 변환

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$